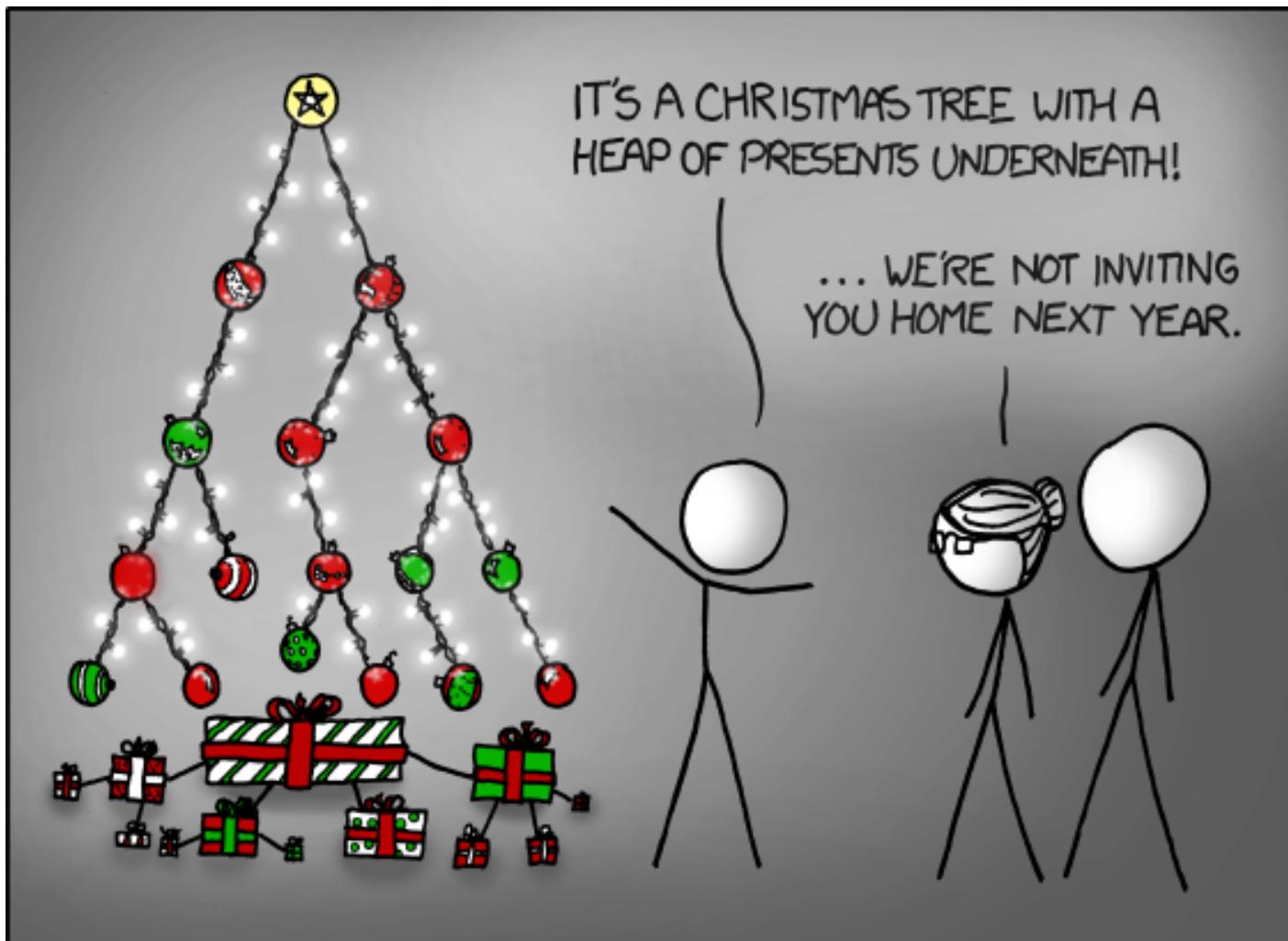


# CSCI 104

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Slides adapted from: Mark Redekopp and David Kempe



<https://xkcd.com/835/>

# SEARCH

# Linear Search

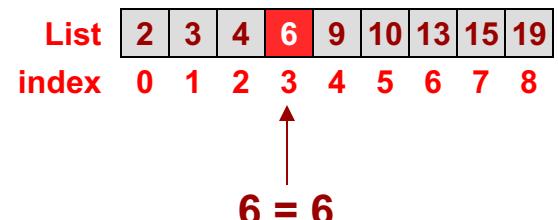
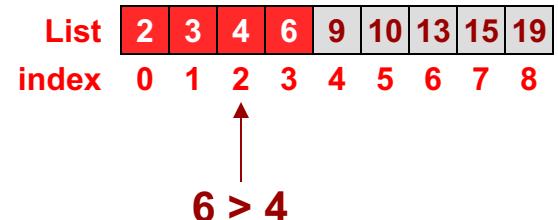
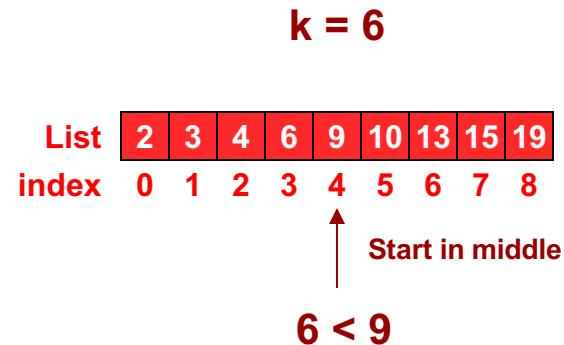
- Search a list (array) for a specific value,  $k$ , and return the location
- Sequential Search
  - Start at first item, check if it is equal to  $k$ , repeat for second, third, fourth item, etc.
- $O(n)$

```
int search(vector<int> mylist, int k)
{
    int i;
    for(i=0; i < mylist.size(); i++) {
        if(mylist[i] == k)
            return i;
    }
    return -1;
}
```

|        |   |   |   |   |   |    |    |    |    |
|--------|---|---|---|---|---|----|----|----|----|
| myList | 2 | 3 | 4 | 6 | 9 | 10 | 13 | 15 | 19 |
| index  | 0 | 1 | 2 | 3 | 4 | 5  | 6  | 7  | 8  |

# Binary Search

- Sequential search does not take advantage of the ordered (a.k.a. sorted) nature of the list
  - Would work the same (equally well) on an ordered or unordered list
- Binary Search
  - Take advantage of ordered list by comparing  $k$  with middle element and based on the result, rule out all numbers greater or smaller, repeat with middle element of remaining list, etc.



# Binary Search

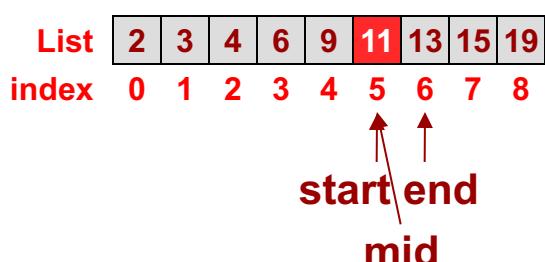
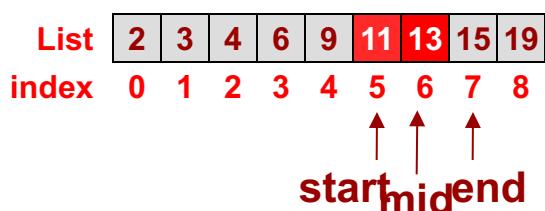
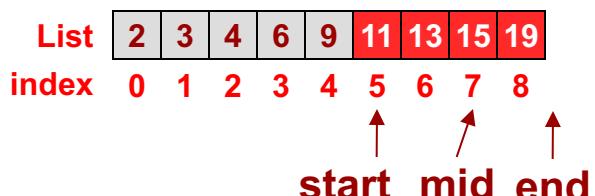
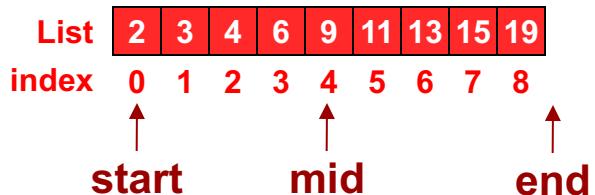
- Search an ordered list (array) for a specific value, k, and return the location
- Binary Search
  - Compare k with middle element of list and if not equal, rule out  $\frac{1}{2}$  of the list and repeat on the other half
  - "Range" Implementations in most languages are [start, end)
  - Start is inclusive, end is non-inclusive (i.e. end will always point to 1 beyond true ending index to make arithmetic work out correctly)

```
int bsearch(vector<int> mylist,
            int k,
            int start, int end)
{
    // range is empty when start == end
    while(start < end) {
        int mid = (start + end)/2;
        if(k == mylist[mid])
            return mid;
        else if(k < mylist[mid])
            end = mid;
        else
            start = mid+1;
    }
    return -1;
}
```

|        |   |   |   |   |   |    |    |    |    |
|--------|---|---|---|---|---|----|----|----|----|
| myList | 2 | 3 | 4 | 6 | 9 | 10 | 13 | 15 | 19 |
| index  | 0 | 1 | 2 | 3 | 4 | 5  | 6  | 7  | 8  |

# Binary Search

**k = 11**



```
int bsearch(vector<int> mylist,
            int k,
            int start, int end)
{
    // range is empty when start == end
    while(start < end) {
        int mid = (start + end)/2;
        if(k == mylist[mid])
            return mid;
        else if(k < mylist[mid])
            end = mid;
        else
            start = mid+1;
    }
    return -1;
}
```

# Search Comparison

- Linear search =  $O(n)$
- Precondition: None
- Works on **ArrayList** or **LinkedList**
- Binary Search =  $O(\log(n))$
- Precondition: **List is sorted**
- Works on **ArrayList** only

```
int search(vector<int> mylist, int k)
{
    int i;
    for(i=0; i < mylist.size(); i++) {
        if(mylist[i] == k)
            return i;
    }
    return -1;
}
```

```
int bsearch(vector<int> mylist,
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    // range is empty when start == end
    while(start < end) {
        int mid = (start + end)/2;
        if(k == mylist[mid])
            return mid;
        else if(k < mylist[mid])
            end = mid;
        else {
            start = mid+1;
        }
    }
    return -1;
}
```

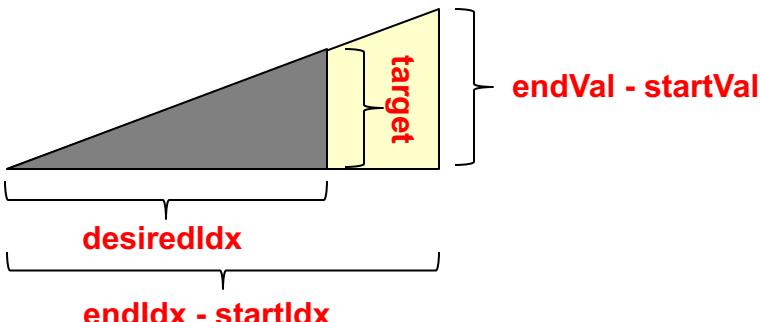
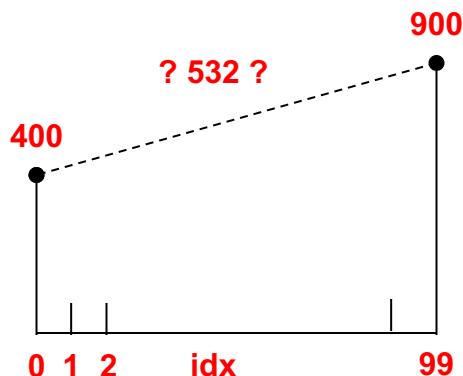
# Introduction to Interpolation Search

- Given a dictionary, if I say look for the word 'banana' would you really do a binary search and start in the middle of the dictionary?
- Assume a uniform distribution of 100 random numbers between [0 and 999]
  - [679 372 554 ... ]
- Now sort them
  - [002 009 015 ... ]
- At what index would you start looking for key=130

|        |     |     |     |     |     |  |     |
|--------|-----|-----|-----|-----|-----|--|-----|
| myList | 002 | 009 | 015 | 024 | 039 |  | 981 |
| index  | 00  | 01  | 02  | 03  | 04  |  | 99  |

# Linear Interpolation

- If I have a range of 100 numbers where the first is 400 and the last is 900, at what index would I expect 532 (my target) to be?



$$\frac{(\text{EndIdx} - \text{StartIdx}+1)}{(\text{EndVal} - \text{StartVal})} = \frac{\text{desiredIdx} - \text{startIdx}}{\text{target} - \text{startVal}}$$

$$(\text{target} - \text{startVal}) * \frac{(\text{EndIdx} - \text{StartIdx}+1)}{(\text{EndVal} - \text{StartVal})} + \text{startIdx} = \text{desiredIdx}$$

$$(532-400)*(100/500) + 0 = \text{desiredIdx}$$

$$132*0.2 = \text{desiredIdx}$$

$$26.4 = \text{desiredIdx}$$

$$\text{floor}(26.4) = 26 = \text{desiredIdx}$$

# Interpolation Search

- Similar to binary search but rather than taking the middle value we compute the interpolated index

```
int bin_search(vector<int> mylist,
               int k,
               int start, int end)
{
    // range is empty when start == end
    while(start < end) {
        int mid = (start + end)/2;
        if(k == mylist[mid])
            return mid;
        else if(k < mylist[mid])
            end = mid;
        else
            start = mid+1;
    }
    return -1;
}
```

```
int interp_search(vector<int> mylist,
                  int k,
                  int start, int end)
{
    // range is empty when start > end
    while(start <= end) {
        int loc =
            interp(mylist, start, end, k);
        if(k == mylist[loc])
            return loc;
        else if(k < mylist[loc])
            start = loc - 1;
        else
            start = loc+1;
    }
    return -1;
}
```

# Another Example

- Suppose we have 1000 doubles in the range 0-1
- Find if 0.7 exists in the list and where
- Use interpolation search
  - First look at location:  $0.7 * 1000 = 700$
  - But when you pick up List[700] you find 0.68
  - We know 0.7 would have to be between location 700 and 100 so we narrow our search to those 300
- Interpolate again for where 0.7 would be in a list of 300 items that start with 0.68 and max value of 1
  - $(0.7-0.68)/(1-0.68) = 0.0675$
  - Interpolated index =  $\text{floor}( 700 + 300*0.0675 ) = 720$
  - You find List[720] = 0.71 so you narrow your search to 700-720
- Interpolate again
  - $(0.7-0.68)/(0.71-0.68) = 0.6667$
  - Interpolated index =  $\text{floor}( 700 + 20*0.6667 ) = 713$

Example from "Y. Perl, A. Itai., and H. Avni, Interpolation Search – A Log Log N Search, Communications of the ACM, Vol. 21, No. 7, July 1978"

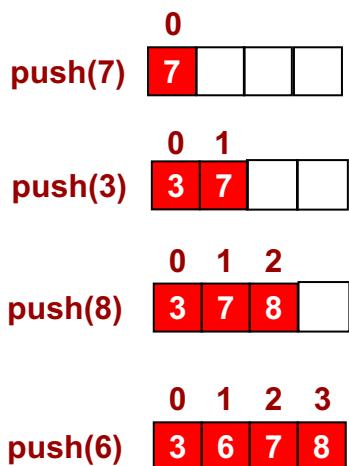
# Interpolation Search Summary

- Requires a sorted list
  - An array list not a linked list (in most cases)
- Binary search =  $O(\log(n))$
- Interpolation search =  $O(\log(\log(n)))$ 
  - If  $n = 1000$ ,  $O(\log(n)) = 10$ ,  $O(\log(\log(n))) = 3.332$
  - If  $n = 256,000$ ,  $O(\log(n)) = 18$ ,  $O(\log(\log(n))) = 4.097$
- Makes an assumption that data is uniformly (linearly) distributed
  - If data is "poorly" distributed (e.g. exponentially, etc.), interpolation search will break down to  $O(\log(n))$  or even  $O(n)$
  - Notice interpolation search uses actual values (target, startVal, endVal) to determine search index
  - Binary search only uses indices (i.e. is data agnostic)
- Assumes some 'distance' metric exists for the data type
  - If we store Webpage what's the distance between two webpages?

# SORTED LISTS

# Overview

- If we need to support fast searching we need sorted data
- Two Options:
  - Sort the unordered list (and keep sorting when we modify it)
  - Keep the list ordered as we modify it
- Now when we insert a value into the list, we'll insert it into the required location to keep the data sorted.
- See example



# Sorted Input Class

- `insert()` puts the value into its correct ordered location
  - Backed by array:  $O( \ )$
  - Backed by `LinkedList`:  $O( \ )$
- `find()` returns the index of the given value
  - Backed by array:  $O( \ )$
  - Backed by `LinkedList`:  $O( \ )$

```
class SortedIntList
{
public:
    bool empty() const;
    int size() const;
    void insert(const int& new_val);
    void remove(int loc);

    // can use binary or interp. search
    int find(int val);

    int& get(int i);
    int const & get(int i) const;
private:
    ???
};
```

# Sorted Input Class

- Assume an array based approach, implement insert()

```
class SortedIntList
{
public:

private:
    int* data; int size; int cap;
};

void SortedIntList::insert(const int& new_val)
{

}

}
```

# XKCD #724

Courtesy of  
Randall Munroe  
@ <http://xkcd.com>

