# CSCI 104

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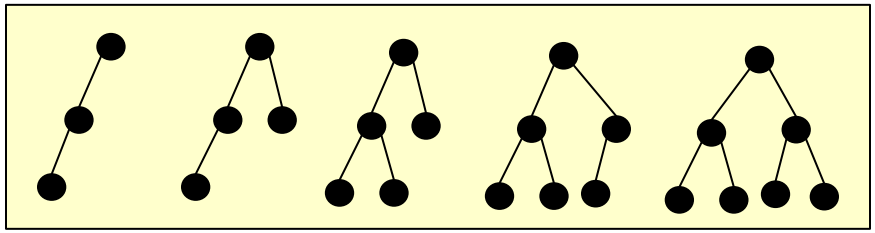
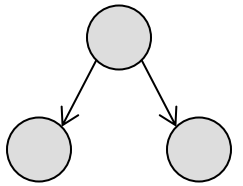
Slides adapted from: Mark Redekopp and David Kempe

# Binary Tree Review

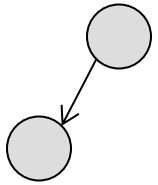
- Full binary tree: Binary tree, T, where
  - If height h>0 and both subtrees are full binary trees of height, h-1
  - If height h==0, then it is full by definition
  - (Tree where all leaves are at level h and all other nodes have 2 children)
- Complete binary tree
  - Tree where levels 0 to h-1 are full and level h is filled from left to right
- Balanced binary tree
  - Tree where subtrees from any node differ in height by at most 1



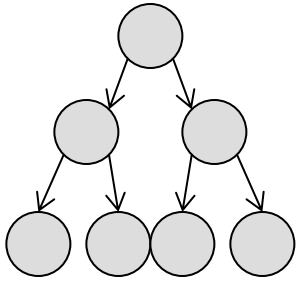
DAPS, 6th Ed. Figure 15-8



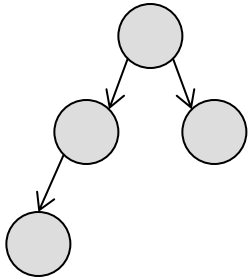
Full



Complete, but not full



Full

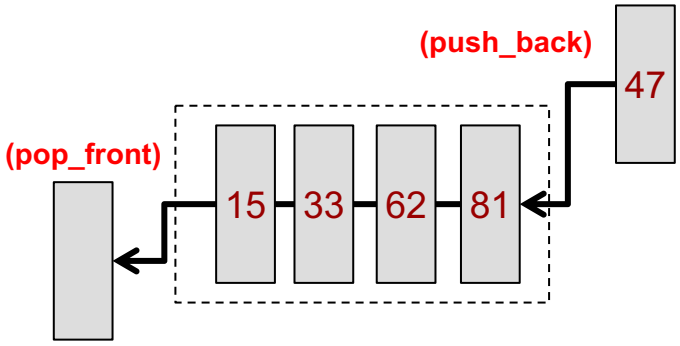


Complete

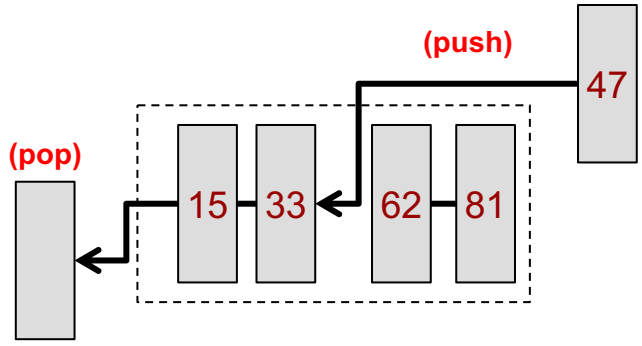
# PRIORITY QUEUES

# Traditional Queue

- Traditional Queues
  - Accesses/orders items based on POSITION (front/back)
  - Did not care about item's VALUE
- Priority Queue
  - Orders items based on VALUE
    - Either minimum or maximum
  - Items arrive in some arbitrary order
  - When removing an item, we always want the minimum or maximum depending on the implementation
    - Heaps that always yield the min value are called min-heaps
    - Heaps that always yield the max value are called max-heaps
  - Leads to a "sorted" list
  - Examples:
    - Think hospital ER, air-traffic control, etc.



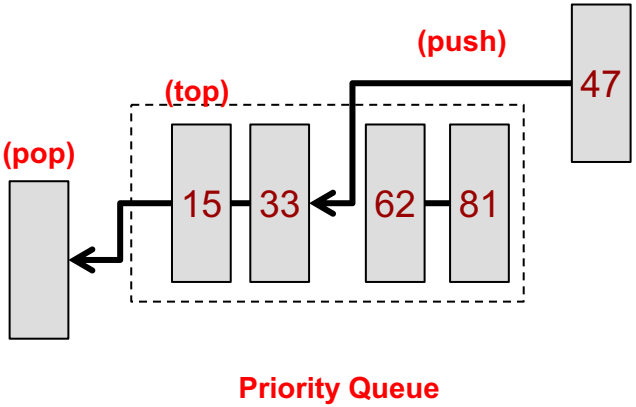

Traditional Queue




Priority Queue

# Priority Queue

- What member functions does a Priority Queue have?
  - push(item) – Add an item to the appropriate location of the PQ
  - top() – Return the min./max. value
  - pop() - Remove the front (min. or max) item from the PQ
  - size() - Number of items in the PQ
  - empty() - Check if the PQ is empty
  - [Optional]: changePriority(item, new_priority)
    - Useful in many algorithms (especially AI and search algorithms)
- Implementations
  - Priority can be based upon intrinsic data-type being stored (i.e. operator<() of type T)
  - Priority can be passed in separately from data type, T,
    - Allows the same object to have different priorities based on the programmer's desire (i.e. same object can be assigned different priorities)



Priority Queue

# Priority Queue Efficiency

- If implemented as a sorted array list
  - Insert() = _____
  - Top() = _____
  - Pop() = _____
- If implemented as an unsorted array list
  - Insert() = _____
  - Top() = _____
  - Pop() = _____

# Priority Queue Efficiency

- If implemented as a sorted array list
  - [Use back of array as location of top element]
  - Insert() = O(n)
  - Top() = O(1)
  - Pop() = O(1)
- If implemented as an unsorted array list
  - Insert() = O(1)
  - Top() = O(n)
  - Pop() = O(n)

# STL Priority Queue

- Implements a max-PQ by default
- Operations:
  - push(new_item)
  - pop(): removes but does not return top item
  - top() return top item (item at back/end of the container)
  - size()
  - empty()
- http://www.cplusplus.com/reference/stl/priority_queue/push/
- Can use Comparator functors to create a min-PQ

```
// priority_queue::push/pop
#include <iostream>
#include <queue>

using namespace std;

int main ()
{
  priority_queue<int> mypq;
  mypq.push(30);
  mypq.push(100);
  mypq.push(25);
  mypq.push(40);
  cout << "Popping out elements...";
  while (!mypq.empty()) {
    cout<< " " << mypq.top();
    mypq.pop();
  }
  cout<< endl;
  return 0;
 }
```

**Code here will print**
**100 40 30 25**

# C++ less and greater

- If your class already has operators < or > and you don't want to write your own functor you can use the C++ built-in functors: less and greater
- Less
  - Compares two objects of type T using the operator< defined for T
- Greater
  - Compares two objects of type T using the operator< defined for T

```
template <typename T>
struct less
{
  bool operator()(const T& v1, const T& v2){
    return v1 < v2;
  }
};

template <typename T>
struct greater
{
  bool operator()(const T& v1, const T& v2){
    return v1 > v2;
  }
};
```

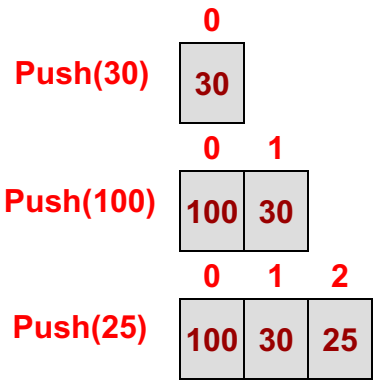
# STL Priority Queue Template

- Template that allows type of element, container class, and comparison operation for ordering to be provided
- First template parameter should be type of element stored
- Second template parameter should be the container class you want to use to store the items (usually vector<type_of_elem>)
- Third template parameters should be comparison functor object/class that will define the order from first to last in the container

```
// priority_queue::push/pop
#include <iostream>
#include <queue>
using namespace std;

int main ()
{ priority_queue<int, vector<int>, greater<int>> mypq;
  mypq.push(30); mypq.push(100); mypq.push(25);
  cout<< "Popping out elements...";
  while (!mypq.empty()) {
    cout<< " " << mypq.top();
    mypq.pop();
  }
}
```

**Code here will print 25, 30, 100**

**greater<int> will yield a min-PQ**
**less<int> will yield a max-PQ**




**Push(n): walk while (item[i] > n), then insert**
**Top(): Return last item**
**Pop(): Remove last item**

# STL Priority Queue Template

- For user defined classes, must implement

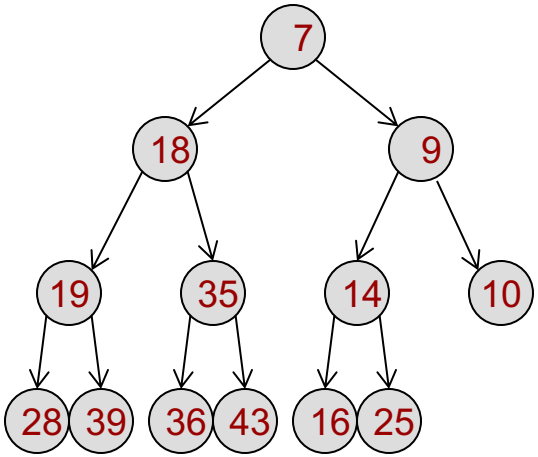
  operator<() for max-heap or operator>() for min-heap
- Code here will pop in order:
  - Jane
  - Charlie
  - Bill

```
// priority_queue::push/pop
#include <iostream>
#include <queue>
#include <string>
using namespace std;

class Item {
 public:
  int score;
  string name;

  Item(int s, string n) { score = s; name = n;}
  bool operator>(const Item &rhs) const {
    if (rhs.score > this->score) {
      return true;
    }
    return false;
  }
};

int main ()
{
  priority_queue<Item, vector<Item>, greater<Item> > mypq;
  Item i1(25,"Bill");    mypq.push(i1);
  Item i2(5,"Jane");     mypq.push(i2);
  Item i3(10,"Charlie"); mypq.push(i3);
  cout<< "Popping out elements...";
  while (!mypq.empty()) {
    cout<< " " << mypq.top().name;
    mypq.pop();
} }
```

# HEAPS

# Heap Data Structure

- Provides an efficient implementation for a priority queue
- Can think of heap as a ***complete*** binary tree that maintains the **heap property**:
  - Heap Property: Every parent is less-than (if min-heap) or greater-than (if max-heap) ***both*** children
  - But no ordering property between children
- Minimum/Maximum value is always the top element




**Min-Heap**

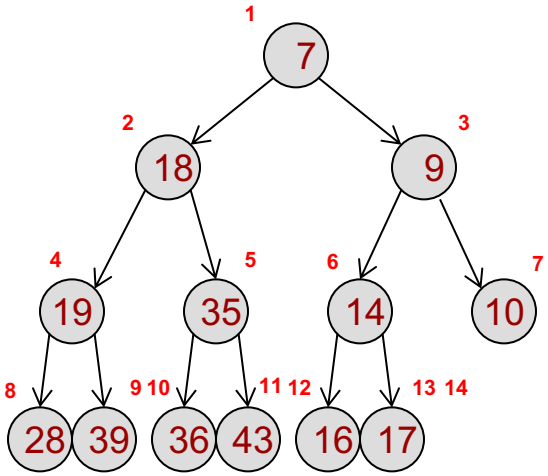
# Heap Operations

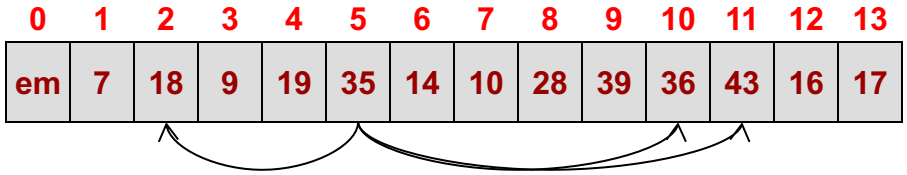
- Push: Add a new item to the heap and modify heap as necessary
- Pop: Remove min/max item and modify heap as necessary
- Top: Returns min/max
- Since heaps are complete binary trees we can use an array/vector as the container

```
template <typename T>
class MinHeap
{
public:
    MinHeap(int init_capacity);
    ~MinHeap()
    void push(const T& item);
    T& top();
    void pop();
    int size() const;
    bool empty() const;

private:
    void heapify(int idx);
    vector<T> items_;
}
```

# Array/Vector Storage for Heap

- Recall: Full binary tree (i.e. only the lowest-level contains empty locations and items added left to right) can be modeled as an array (let's say it starts at index 1) where:
  - Parent(i) = i/2
  - Left_child(p) = 2*p
  - Right_child(p) = 2*p + 1

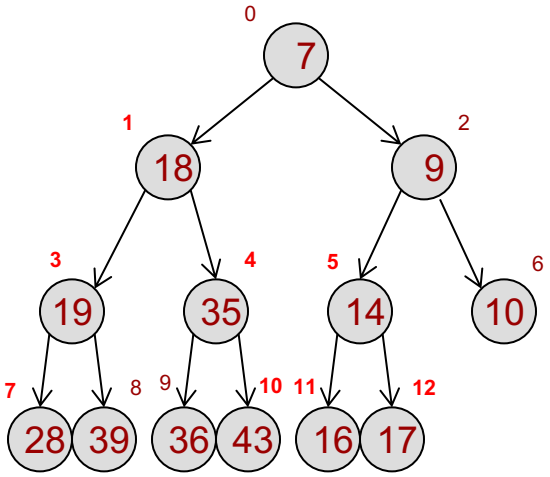


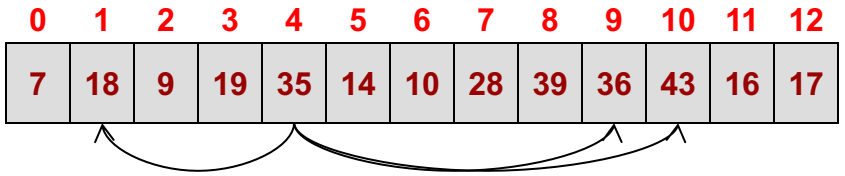




parent(5) = 5/2 = 2
Left_child(5) = 2*5 = 10
Right_child(5) = 2*5+1 = 11

# Array/Vector Storage for Heap

- We can also use 0-based indexing
  - Parent(i) = (i-1)/2
  - Left_child(p) = 2*p+1
  - Right_child(p) = 2*p + 2

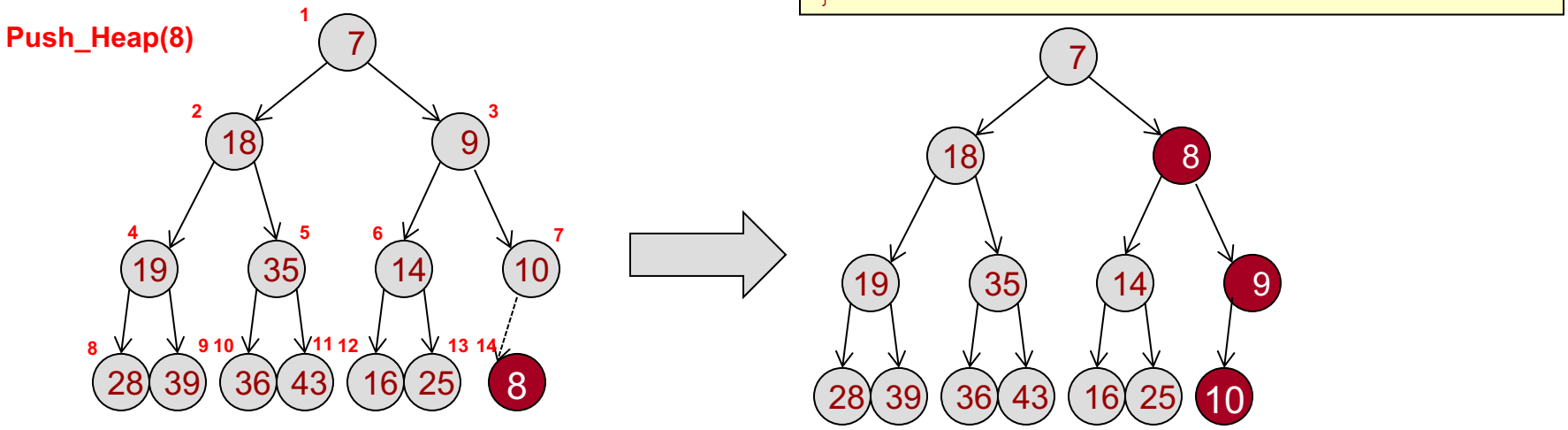


# Push Heap / TrickleUp

- Add item to first free location at bottom of tree
- Recursively promote it up while it is less than its parent
  - Remember valid heap all parents < children…so we need to promote it up until that property is satisfied

```
void MinHeap<T>::push(const T& item)
{
  items_.push_back(item);
  trickleUp(items_.size()-1);
}

void trickleUp(int loc)
{
  // could be implemented recursively
  int parent = loc/2;
  while(parent >= 1 &&
        items_[loc] < items_[parent] )
  {  swap(items_[parent], items_[loc]);
     loc = parent;
     parent = loc/2;
  }
}
```

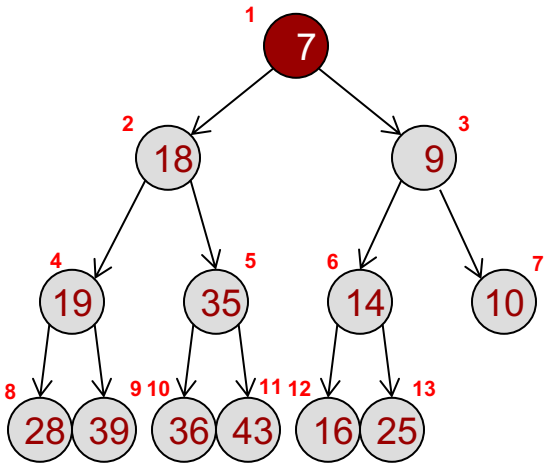


# Top()

- Top() simply needs to return first item

```
T& MinHeap<T>::top()
{
  if( empty() )
    throw(std::out_of_range());
  return items_[1];
}
```

Top() returns 7

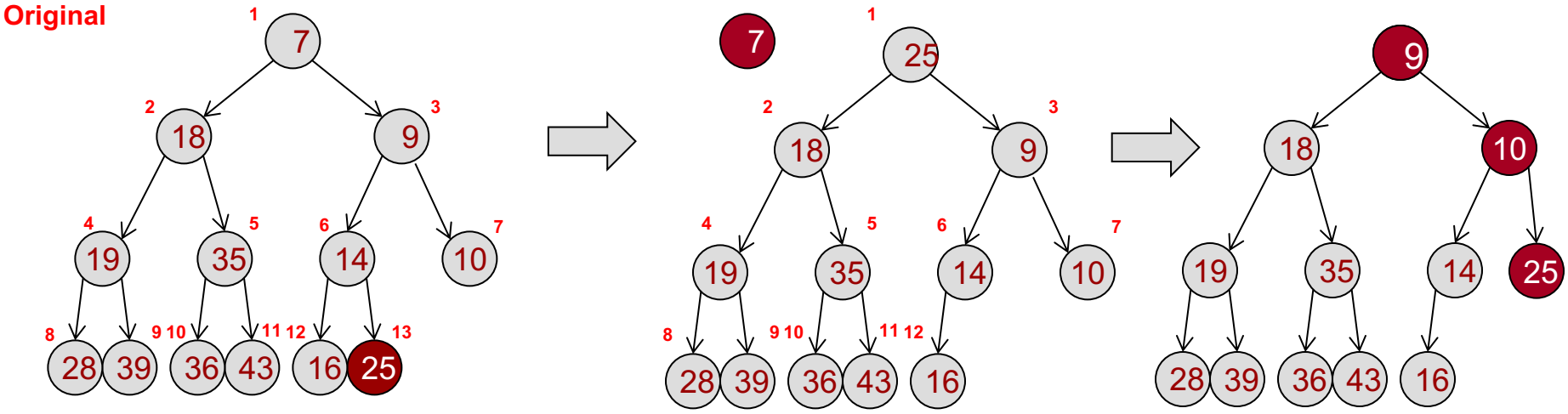


# Pop Heap / Heapify (TrickleDown)

- Pop utilizes the "heapify" algorithm (a.k.a. trickleDown)
- Takes last (greatest) node puts it in the top location and then recursively swaps it for the smallest child until it is in its right place

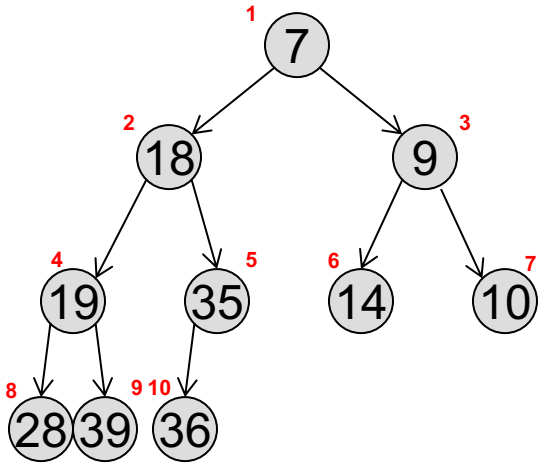
```
void MinHeap<T>::pop()
{ items_[1] = items_.back(); items_.pop_back()
  heapify(1); // a.k.a. trickleDown()
}
```

```
void MinHeap<T>::heapify(int idx)
{
  if(idx == leaf node) return;
  int smallerChild = 2*idx; // start w/ left
  if(right child exists) {
    int rChild = smallerChild+1;
    if(items_[rChild] < items_[smallerChild])
       smallerChild = rChild;
  } }
  if(items_[idx] > items_[smallerChild]){
       swap(items_[idx], items_[smallerChild]);
       heapify(smallerChild);
} }
```

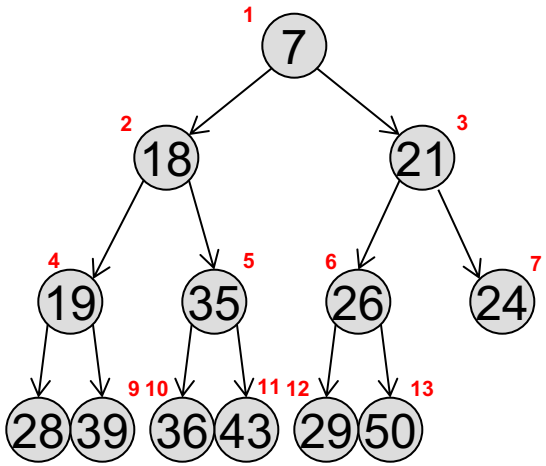


# Practice

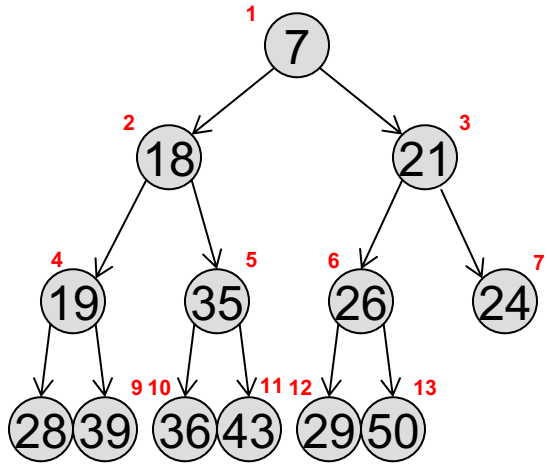
**Push(11)**



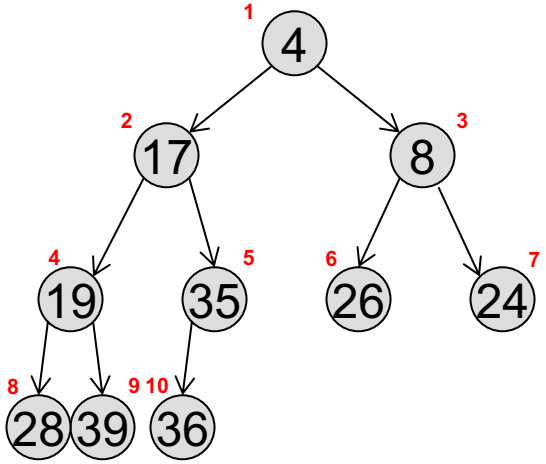

**Push(23)**




**Pop()**




**Pop()**



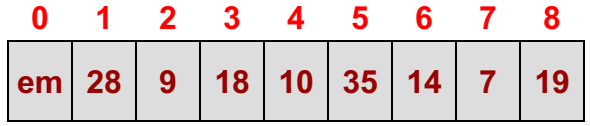
Building a heap out of an array

# HEAPSORT

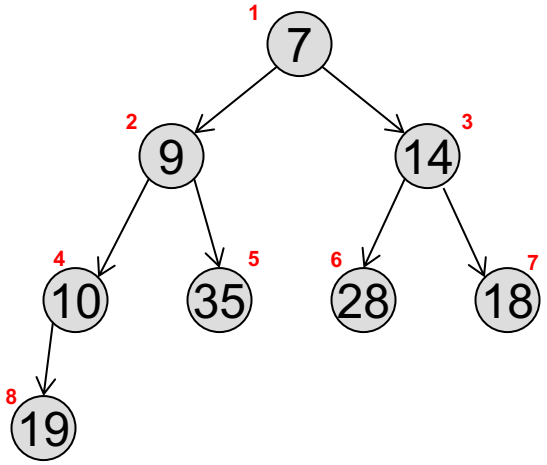
# Using a Heap to Sort

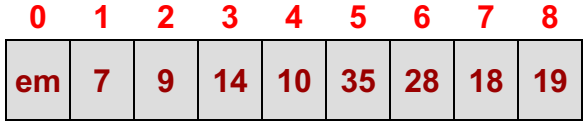
- If we could make a valid heap out of an arbitrary array, could we use that heap to ***sort*** our data?
- Sure, just call top() and pop() ***n*** times
  - You'll get your data out in sorted order
- How long would that take?
  - **n** calls to top() and pop()
  - top() = O(1)
  - pop = O(log n)
- Thus total time = O(n * log n)

- But how long does it take to convert the array to a valid heap?

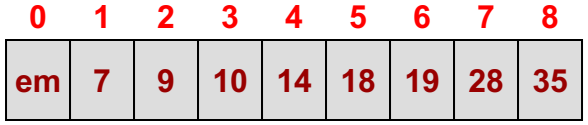


Arbitrary Array



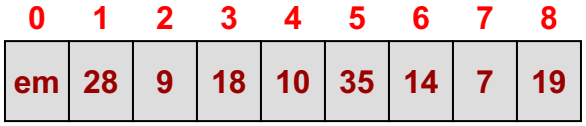
Array Converted to Valid Heap



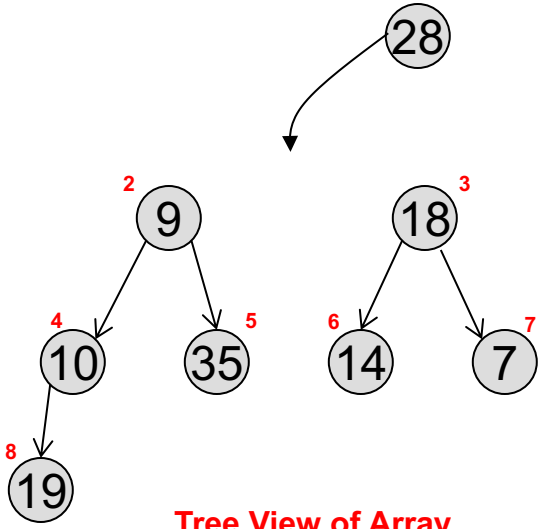
Array after top/popping the heap n times

# Converting An Array to a Heap

- If we have two heaps can we unify them with some arbitrary value
- If we put an arbitrary value in the top spot how can we make a heap?
- Heapify!! (we did this in pop() )



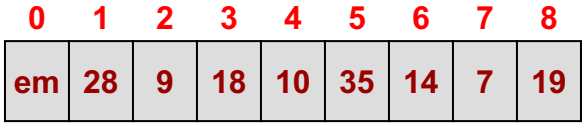

Original Array



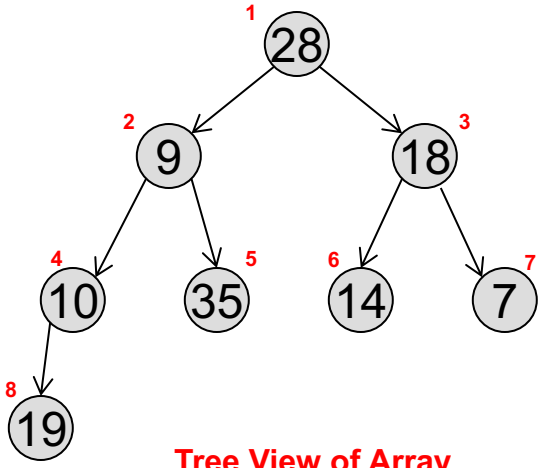

Tree View of Array

# Converting An Array to a Heap

- To convert an array to a heap we can use the idea of first making heaps of both sub-trees and then combining the sub-trees (a.k.a. semi heaps) into one unified heap by calling heapify() on their parent()



Original Array

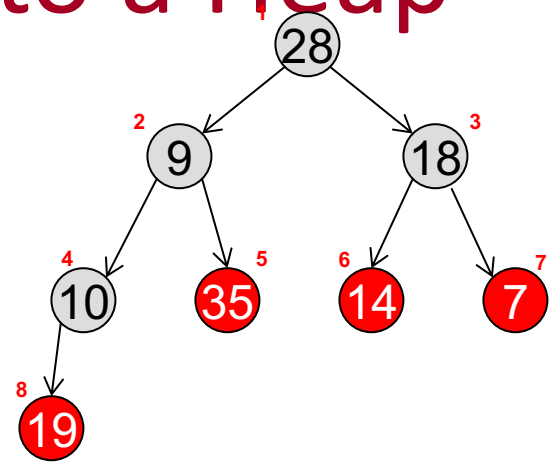


Tree View of Array

- First consider all leaf nodes, are they valid heaps if you think of them as the root of a tree?
  - Yes!!
- So just start at the first non-leaf

# Converting An Array to a Heap

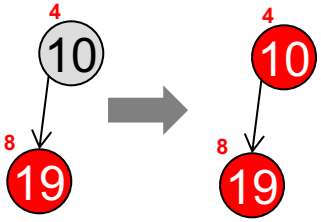
- First consider all leaf nodes, are they valid heaps if you think of them as the root of a tree?
  - Yes!!
- So just start at the first non-leaf
  - Heapify(Loc. 4)




Leafs are valid heaps by definition

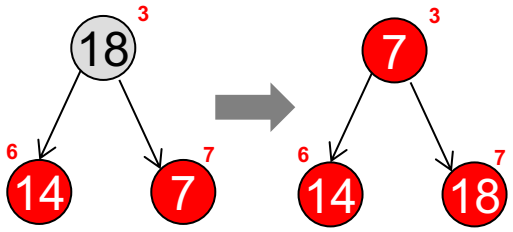
**heapify(4)**

Already in the right order



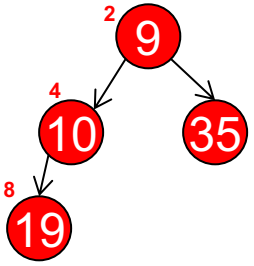

**heapify(3)**

Swap 18 & 7



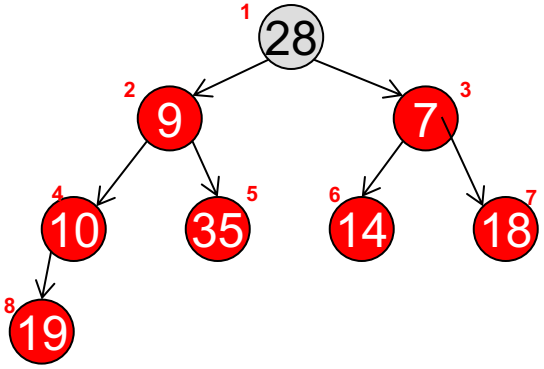

**heapify(2)**

Already a heap




**heapify(1)**

Swap 28 <-> 7
Swap 28 <-> 14

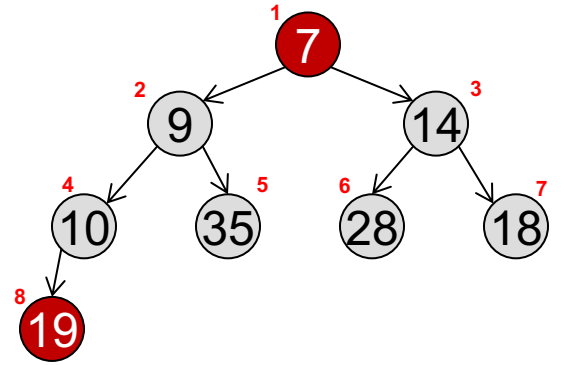


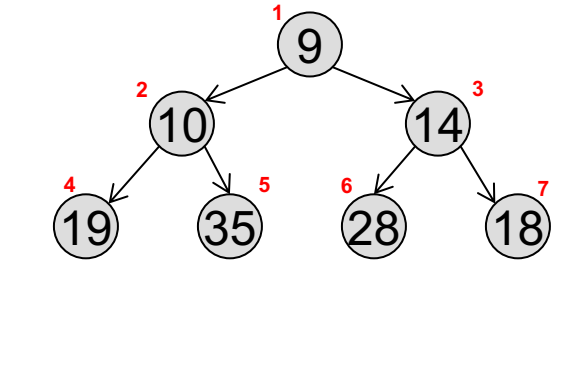
# Converting An Array to a Heap

- Now that we have a valid heap, we can sort by top and popping…
- Can we do it in place?
  - Yes, Break the array into "heap" and "sorted" areas, iteratively adding to the "sorted" area

Swap top & last

heapify(1)



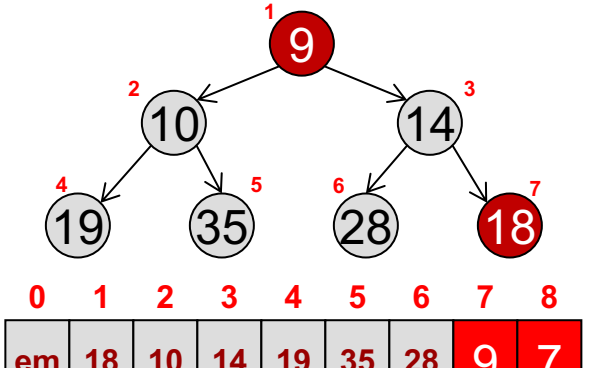



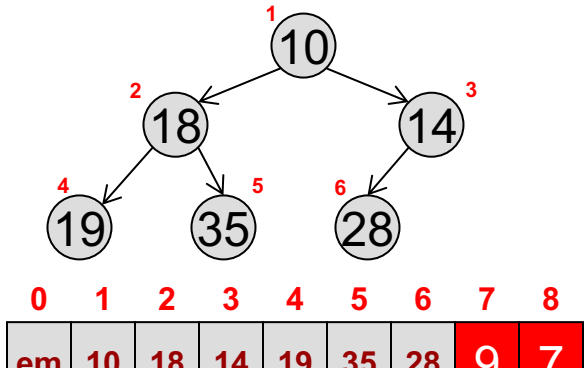

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| em | 19 | 9 | 14 | 10 | 35 | 28 | 18 | 7 |

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| em | 9 | 10 | 14 | 19 | 35 | 28 | 18 | 7 |

Swap top & last

heapify(1)




| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| em | 18 | 10 | 14 | 19 | 35 | 28 | 9 | 7 |

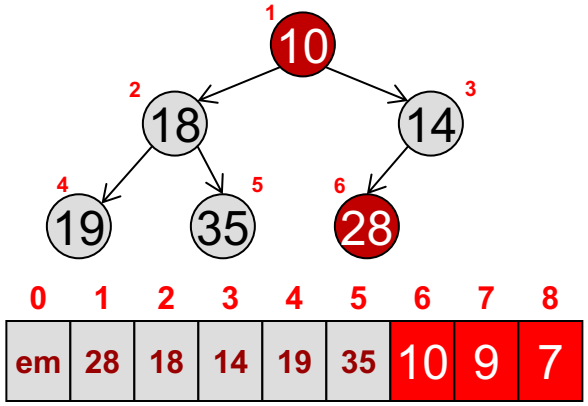
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| em | 10 | 18 | 14 | 19 | 35 | 28 | 9 | 7 |

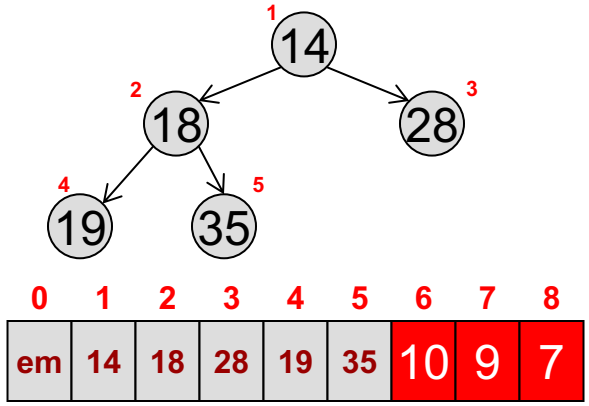
# Converting An Array to a Heap

- Now that we have a valid heap, we can sort by top and popping…
- Can we do it in place?
  - Yes, Break the array into "heap" and "sorted" areas, iteratively adding to the "sorted" area

Swap top & last

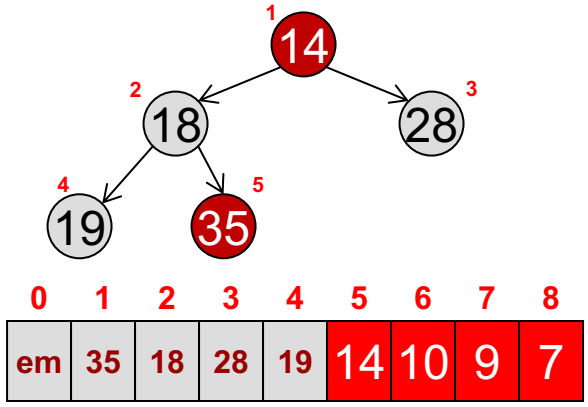
heapify(1)

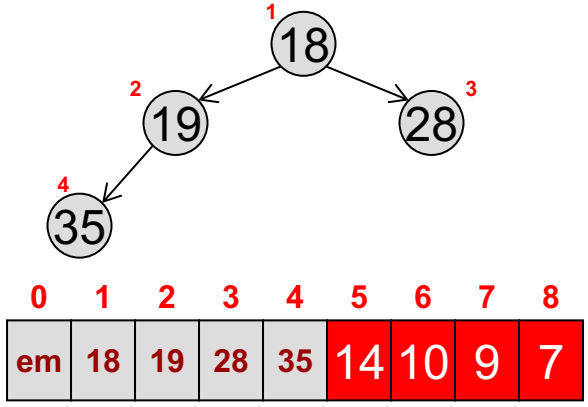


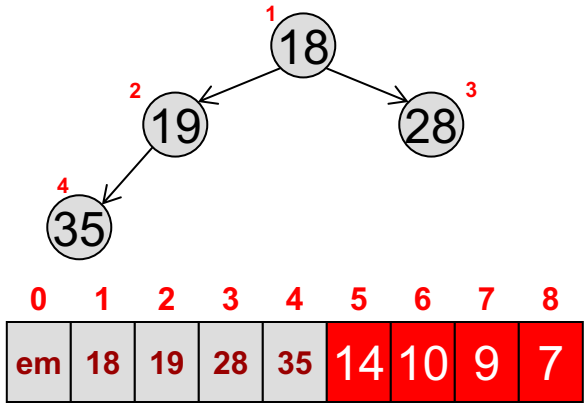

Swap top & last

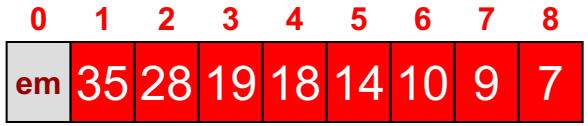
heapify(1)



# Converting An Array to a Heap




- Notice the result is in descending order.
- How could we make it ascending order?
  - Create a max heap rather than min heap.

# Build-Heap Run-Time

- To build a heap from an arbitrary array require n calls to heapify.
- Heapify take O(_____)
- Let's be more specific:
  - Heapify takes O(h)
  - Because most of the heapify calls are made in the bottom of the tree (shallow h), it turns out heapify can be done in O(n)